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Assignment 3.2

Assignment: Section 2.2: 2, 4, 12, 16, 18, 20 (7th edition)

2.

a) A ∩ B

b) A – B

c) A ∪ B

d) ∪

4.

a) {a, b, c, d, e, f, g, h}

b) {a, b, c, d, e}

c) *ϕ*

d) {f, g, h}

12.

Let's prove A U (A ∩ B) ⊆ A : Let's assume x ∈ A U(A ∩ B), then there are three options: x ∈ A, x ∈ (A ∩ B), x ∈ A and x ∈ (A ∩ B). In the case of the first option, x ∈ A, we immediately get that A U (A ∩ B) ⊆ A, In the case of the third option, we also get A U (A ∩ B) ⊆ A immediately, and in the case of the second option, x ∈ (A ∩ B), by definition of intersection of two sets, we get that x ∈ A and x ∈ B, which again confirms that A U (A ∩ B) ⊆ A.

16.

a) For x ∈ (A ∩ B), x ∈ A and x ∈ B. So, x ∈ A. Since x ∈ (A ∩ B) ⇒ x ∈ A, we have (A ∩ B) ⊆ A.

b) Suppose x ∈ A. Then certainly (x ∈ A) ∨ (x ∈ B). So, x ∈ A ∪ B. Since x ∈ A ⇒ x ∈ A ∪ B, we have A ⊆ A ∪ B

c) Suppose x ∈ A\B. Then (x ∈ A)∧(x 6∈ B). So x ∈ A. Thus (x ∈ (A\B)) ⇒ x ∈ A, so A \ B ⊆ A.

d) Suppose x ∈ A ∩ (B \ A). So x ∈ A and x ∈ B \ A. Since x ∈ B \ A, x ∈ B and x 6∈ A. But x ∈ A! Contradiction, so our assumption x ∈ A ∩ (B \ A) must be wrong. Thus A ∩ (B \ A) must be empty.

e) Suppose x ∈ A ∪ (B \ A). Then x ∈ A or x ∈ B \ A. If x ∈ B \ A, (x ∈ B) ∨ (x 6∈ A), so x ∈ B. So x ∈ A or x ∈ B, which implies x ∈ A ∪ B. Thus, x ∈ A ∪ (B \ A) ⇒ x ∈ A ∪ B, or A ∪ (B \ A) ⊆ A ∪ B.

18.

a) Let’s say x ∈ (A ∪ B). By the definition of union, x ∈ A or x ∈ B. It follows

that x ∈ A, x ∈ B, or x ∈ C holds. Using the definition of union, we conclude that

x ∈ (A ∪ B ∪ C).

b) Let’s say x ∈ (A ∩ B ∩ C). By the definition of intersection, x ∈ A, x ∈ B, and

x ∈ C. Therefore, it is apparent that x ∈ A and x ∈ B holds. By the definition of intersection,

x ∈ A ∩ B. So we conclude that (A ∩ B ∩ C) ⊆ (A ∩ B).

c) Let’s say x ∈ (A − B) − C. By the definition of difference, x ∈ A, x ~~∈~~ B, and x ~~∈~~ C. Therefore, x ∈ A and x ~~∈~~ C. By the definition of difference, x ∈ A−C.

d) Let’s say that (*A − C*) *∩* (*C − B*) *=* *ϕ*. Then there exits an *x* such that *x ∈* (*A − C*) *∩* (*C − B*). By the definition of intersection, *x ∈* (*A − C*) and *x ∈* (*C − B*). By the definition of difference, it follows that *x ∈ A* and *x ~~∈~~ C* and that *x ∈ C* and *x ~~∈~~ B*. This leads to the contradiction that *x ∈ C* and *x ~~∈~~ C*. Hence, our assumption is false and we conclude that (*A − C*) *∩* (*C − B*) = *ϕ*.

e)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| A B C | C --- A | *B − A* | *B ∪ C* | (*C − A*) *∪* (*B − A*) | (*B ∪ C*) *− A* |
| 1 1 1 | 0 | 0 | 1 | 0 | 0 |
| 1 1 0 | 0 | 0 | 1 | 0 | 0 |
| 1 0 1 | 0 | 0 | 1 | 0 | 0 |
| 1 0 0 | 0 | 0 | 0 | 0 | 0 |
| 0 1 1 | 1 | 1 | 1 | 1 | 1 |
| 0 1 0 | 0 | 1 | 1 | 1 | 1 |
| 0 0 1 | 1 | 0 | 1 | 1 | 1 |
| 0 0 0 | 0 | 0 | 0 | 0 | 0 |

Membership table proves identity.

20.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| A B | (*A ∩ B*) |  | (*A ∩* ) | (*A ∩ B*) *∪* (*A ∩* ) |
| 1 1 | 1 | 0 | 0 | 1 |
| 1 0 | 0 | 1 | 1 | 1 |
| 0 1 | 0 | 0 | 0 | 0 |
| 0 0 | 0 | 1 | 0 | 0 |

Membership table proves identity